Choosing a Spectral Measurement for Data Analysis

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This month's column was suppose to address which signal processing window you should use to analyze data. I have decided to make this a two part question by first addressing what type of spectral measurement you should use. Then, next month I will address the issue of signal processing windows since window selection depends largely on the type of data being analyzed.

Question: **What type of frequency spectrum measurement should I use to analyze my data?**

Answer: The answer to this question depends upon the type of data you are trying to analyze. Let's assume you are analyzing operating data from a device that is causing a noise and/or vibration problem. You would like to analyze the operating data to quantify the levels and frequency content of the noise and/or vibration in terms of its frequency spectrum. The scaling or units you should use for the analyzed spectrum depends on the nature of the signal being analyzed. Generally, a signal can be classified into three distinct types: Deterministic, Random or Transient.

When the signal is Deterministic, the spectrum units should be Root Mean Square (RMS) or Mean Square (Power). When the signal is Random, the appropriate unit is Power Spectral Density (PSD) or Spectral Density (SD). And, when transients are analyzed, you should scale the measurement as Energy Spectral Density (ESD).

Signals that we analyze can be characterized into two main groups, stationary signals and nonstationary signals. A stationary signal is a set of time histories having statistical properties that are invariant with respect to translations in time. Whereas the statistical properties of nonstationary signals do vary over time. When dealing with nonstationary signals, we typically use short time frames to analyze signal magnitudes and display the results in a three-dimensional waterfall or colored spectrogram. We will restrict our discussion to the analysis of stationary signals.

Deterministic Signals. Stationary deterministic signals are entirely made up of sinusoids at discrete frequencies. The frequency analysis resolution is determined by the filter bandwidth of the analyzer being used for analysis. Ideally, the filter bandwidth (analyzer's frequency resolution) should be made small enough to contain only one sinusoid at any given time. This would enable the analyzer to distinguish between the two most closely spaced frequency components. In this situation, the power transmitted by the filter is independent of the analysis bandwidth because all the power is at a discrete frequency. Because of this, the averaged frequency spectrum should be appropriately scaled in terms of mean squared amplitude (Power) and would have units of EU^2, where EU is the calibrated engineering units (EU) setup for the measurement. Or, the measurement may also be scaled as the root mean square amplitude (RMS). Figure 1 illustrates a time record and frequency spectrum for a stationary deterministic signal.

Random Signals. Continuous, stationary random signals have spectra, which are continuous in the frequency domain. For this reason, there is a continuous frequency distribution within the filter passband. Therefore, the power transmitted by the filter depends on its bandwidth, i.e., the resolution of the analyzer. By dividing the transmitted power by the filter bandwidth, we can eliminate its influence. This normalizes the spectrum to a mean square spectral density, often referred to as the Power Spectral Density (PSD) which is a measure of the power per unit bandwidth. The units for this type of measurement are expressed in EU^2/Hz or the Spectral Density (SD), which is the square root of this value and is expressed in $EU/\sqrt{Hz}$. Figure 2 illustrates a typical time history and averaged Power Spectral Density function for a stationary random signal.

Transient Signals. A transient signal starts and finishes at zero amplitude. This type of signal contains a finite amount of energy due to its finite length. Therefore, a transient cannot be characterized in terms of power. The spectral amplitude of a transient signal depends on the length of the time record. So with a fixed duration transient signal, the longer the time record becomes the lower the average amplitude. Since transients also have a spectrum continuously distributed over frequency, the transmitted power must be normalized with respect to the analyzer's filter bandwidth, just as with a PSD. But, the measurement must be additionally rescaled according to the record length. Thus, appropriate units for the spectrum of a transient are energy per unit bandwidth, which is referred to as the Energy Spectral Density (ESD) with units of EU^2-sec/Hz. Figure 3 illustrates a typical transient time history and associated energy spectral density spectrum.

The above three signal-type definitions will help you decide on the appropriate units to use when measuring frequency spectra in a lot of instances. But what do you do when you have a signal that is comprised of a combination of signal types such as a deterministic signal and a random signal? An example of this might be an acceleration measurement made on a piece of avionics gear in a helicopter. The measured acceleration could be comprised of broadband random due to turbulence and narrowband sine (deterministic) due to the rotors. In this situation, the amplitude of the deterministic part of the signal must be measured using a spectrum with units of mean squared amplitude (Power) or root mean square amplitude (RMS) to correctly measure the amplitude. Whereas, the amplitude of the broadband random part must be displayed with Power Spectral Density (PSD) or Spectral Density (SD) units in order to measure the correct amplitude. Thus, there is a problem since one of the components in the spectrum will have an incorrect magnitude depending on which units you choose to use for the spectrum. If a Power Spectrum measurement is used, the amplitudes of the discrete frequency components will be correct but the amplitude of the broadband random will be a function of the analyzers bandwidth, i.e., resolution. Conversely, if one chooses to use a Power Spectral Density measurement, the amplitude of the broadband random signal will be correct however the amplitude of the discrete components will now be a function of the analyzer's bandwidth. So in this case one cannot simultaneously measure the amplitudes of the deterministic and random signals within the same measurement unless the bandwidth used by the analyzer is 1 Hz. And, in this case the Power Spectrum and Power Spectral Density measurements will be the same.

Figures 4 and 5 illustrate the effects of analyzer filter bandwidth (resolution) on the spectral measurement of a signal containing both deterministic and random signal types. Figure 4 shows the RMS Spectrum of the signal using two different analyzer resolutions that differed by a factor of 16. Notice that the amplitude of the sine (deterministic) component is independent of the analyzer resolution. However, the amplitude of the broadband random part is now different by a factor of 4 because of the difference in resolution. Figure 5 illustrates the same signals using a Spectral Density (SD) measurement. Now the broadband random part measured with the two different analyzer resolutions is the same amplitude but the amplitudes of the sine component now differ by a factor of 4.

So, which measure measurement is correct? Well it depends on what you want to measure. If you want the correct amplitudes of the deterministic components, you should use a RMS Spectrum or Power Spectrum measurement. And if you want the correct amplitude of the broadband random part, then you need to use a Power Spectral Density (PSD) or Spectral Density (SD) measurement. Regardless of the specific measurement, you should always label them with the units, analyzer resolution and the type of signal processing window used (selection of signal processing windows will be discussed in next months column).

Next Month's Question: Which Signal Processing Window should I be using to analyze my data?

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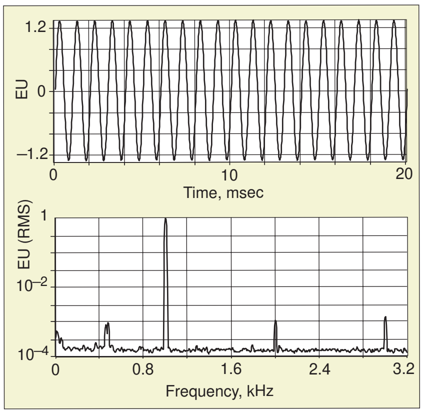

Figure 1. Stationary deterministic signal – time history and RMS spectrum.

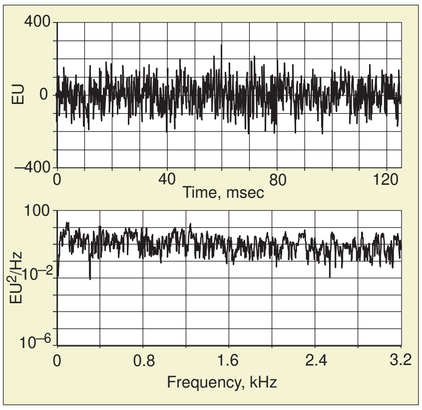

Figure 2. Stationary random signal – time history and PSD spectrum.

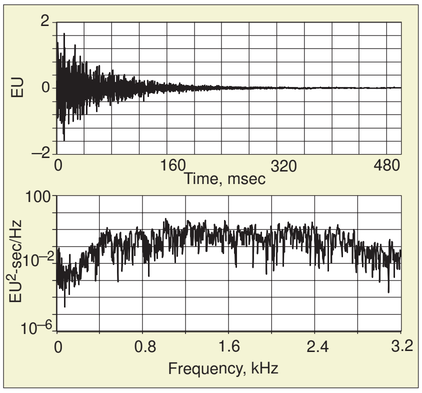

Figure 3. Transient signal – time history and ESD spectrum.

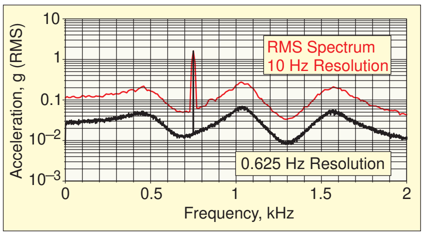

Figure 4. Root mean squared (RMS) spectrum.

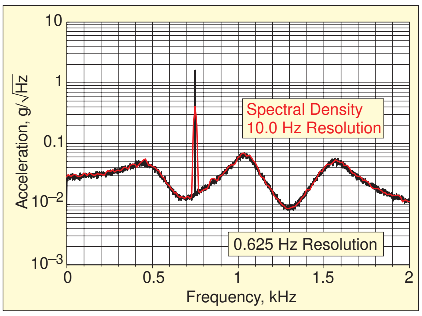

Figure 5. Mean squared spectrum (spectral density).